Udledninger til Hjemmeopgave 3, 2001E, opgave 3.6

Virksomhedens problem:

$$\max_{x_1} \Pi \Rightarrow \frac{\partial \Pi}{\partial x_1} = 0$$

$$\Rightarrow p \cdot MP_1 = w_1$$

Vi indsætter MP_1 fra 3.1:

$$\Rightarrow p \cdot \alpha\beta x_1^{\beta-1} \bar{x}_2^{\gamma} \bar{x}_3^{\delta} = w_1$$

$$\Leftrightarrow x_1^{\beta-1} = w_1 \cdot \left(p\alpha\beta \bar{x}_2^{\gamma} \bar{x}_3^{\delta}\right)^{-1}$$

$$\Leftrightarrow x_1^* = w_1^{\frac{1}{\beta-1}} \left(p\alpha\beta \bar{x}_2^{\gamma} \bar{x}_3^{\delta}\right)^{-\frac{1}{\beta-1}}$$

$$\Leftrightarrow x_1^* = \left(\frac{p\alpha\beta}{w_1} \bar{x}_2^{\gamma} \bar{x}_3^{\delta}\right)^{-\frac{1}{\beta-1}}$$

$$\Leftrightarrow x_1^* = \left(\frac{p\alpha\beta}{w_1} \bar{x}_2^{\gamma} \bar{x}_3^{\delta}\right)^{\frac{1}{1-\beta}}, \text{ da } -\frac{1}{\beta-1} = \frac{1}{1-\beta}.$$

Vi mangler at finde y^*. Det gøres ved at indsætte x_1^* i produktionsfunktionen:

$$y^* = \alpha \cdot \left[\left(\frac{p\alpha\beta}{w_1} \bar{x}_2^{\gamma} \bar{x}_3^{\delta}\right)^{\frac{1}{1-\beta}}\right]^{\beta} \bar{x}_2^{\gamma} \bar{x}_3^{\delta} = \alpha \cdot \left(\frac{p\alpha\beta}{w_1} \bar{x}_2^{\gamma} \bar{x}_3^{\delta}\right)^{\frac{\beta}{1-\beta}} \bar{x}_2^{\gamma} \bar{x}_3^{\delta}$$

$$\Leftrightarrow y^* = \alpha\alpha^{\frac{\beta}{1-\beta}} \left(\frac{p\beta}{w_1}\right)^{\frac{\beta}{1-\beta}} \bar{x}_2^{\gamma} \bar{x}_2^{\frac{\gamma\beta}{1-\beta}} \bar{x}_3^{\delta} \bar{x}_3^{\frac{\delta\beta}{1-\beta}} = \alpha^{\frac{1-\beta+\beta}{1-\beta}} \left(\frac{p\beta}{w_1}\right)^{\frac{\beta}{1-\beta}} \bar{x}_2^{\frac{\gamma(1-\beta)}{1-\beta}+\frac{\gamma\beta}{1-\beta}} \bar{x}_3^{\frac{\delta(1-\beta)}{1-\beta}+\frac{\delta\beta}{1-\beta}}$$

$$y^* = \alpha^{\frac{1}{1-\beta}} \left(\frac{p\beta}{w_1}\right)^{\frac{\beta}{1-\beta}} \bar{x}_2^{\frac{\gamma}{1-\beta}} \bar{x}_3^{\frac{\delta}{1-\beta}}$$