DEPARTMENT OF ECONOMICS
UNIVERSITY OF COPENHAGEN

Econometrics 2, Class 1

Problem Set #9
November 14, 2005

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Problem Set #9

- The first two exercises use Monte Carlo simulations to demonstrate some properties of the Dickey-Fuller unit root test.
- The last exercise tests what you have learnt with some true/false questions.
- Last time I promised we would start on cointegration, but this is postponed to next week…

Repetition: Rejection frequency, size and power

Rejection frequency: Proportion of cases where a hypothesis is rejected.

$\mathcal{H}_2 \; : \; \beta_0 = 0$ TRUE. Rejection frequency $\rightarrow 5\%$

$\mathcal{H}_3 \; : \; \beta_1 = 0$ FALSE. Rejection frequency $\rightarrow 100\%$

as $T \rightarrow \infty$

Rejecting H_2 is an example of a TYPE I error (rejecting a true null hypothesis). Probability is controlled by researcher through choice of significance level: the ***size*** of the test. As T increases, we will become less and less likely to reject the true hypothesis.

Accepting H_3 is an example of a TYPE II error (not rejecting the null when the alternative is true). Depends on the true parameter values. If hypothesis is very different from the true value, then the probability of rejection will be large. The reverse, the probability of rejecting the null when it is false, is the ***power*** of the test. As T increases, we will become more and more likely to reject the false hypothesis.

9.1 Size and power of the DF test

This exercise considers the size and power properties of the Dickey-Fuller test for a unit root. We assume that a data set, $y_1, y_2, ..., y_T$, is generated from a data generating process (DGP) given by

$$y_t = \theta \cdot y_{t-1} + u_t, \quad u_t \sim N(0,1), \tag{9.1}$$

where the initial value is given, $y_0 = 0$. We want to analyze the regression model

$$y_t = \beta_0 + \beta_1 \cdot y_{t-1} + \epsilon_t, \tag{9.2}$$

in particular the properties of the Dickey Fuller $t-$test for the unit root hypothesis against a stationary alternative, i.e.

$$H_0 : \beta_1 = 1 \quad \text{against} \quad H_A : |\beta_1| < 1.$$

In each replication, $m = 1, ..., M$, we can either reject or accept the hypothesis H_0. Recall that the proportion of cases where the hypothesis is rejected is denoted *the rejection frequency*.

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(1) Rejection frequencies, size and power

- If there is a unit root, then the Dickey-Fuller null hypothesis is true.
- We would thus expect a rejection frequency of 5% asymptotically. (*Size*)
- If there is not a unit root, and the null is thus false, then we would expect the rejection frequency to approach 100%.
- It turns out that the *power* of the DF test (rejecting the null when it is false) is not very good. This is because any very persistent process could (falsely) appear to have a unit root, thus leading to an accept of the (false) null.

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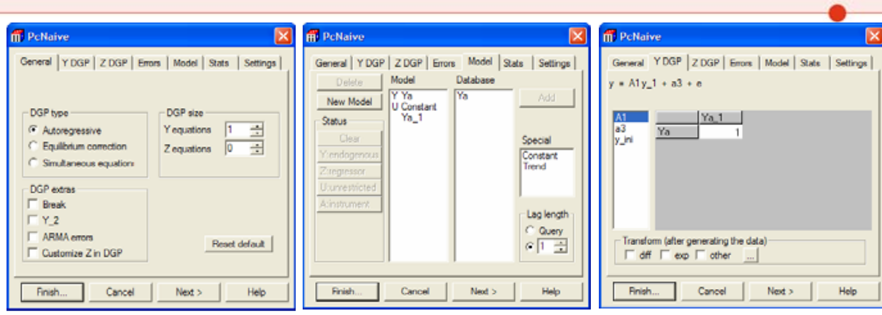
(2) PcNaive DGP...
How to specify the DGP



Autoregressive parameter, A1 = 1

Intercept, a3 = 0

Initial value, y_ini = 0

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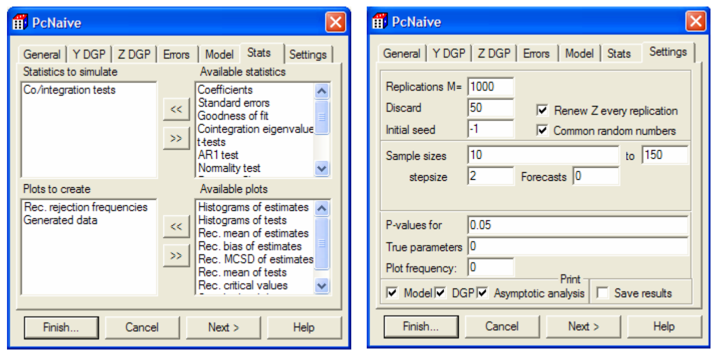
How to specify the DGP (cont.)



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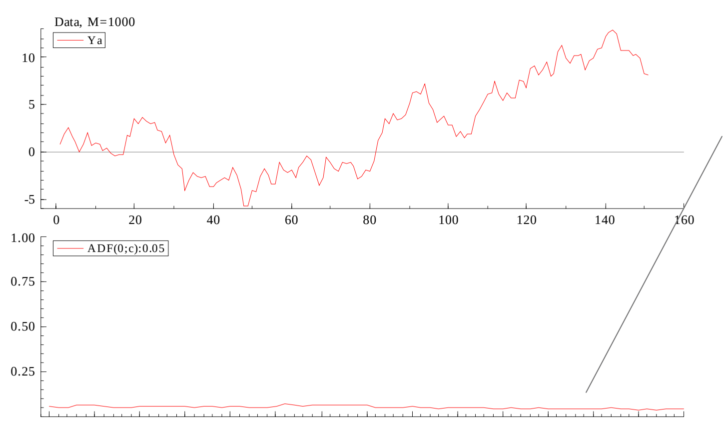
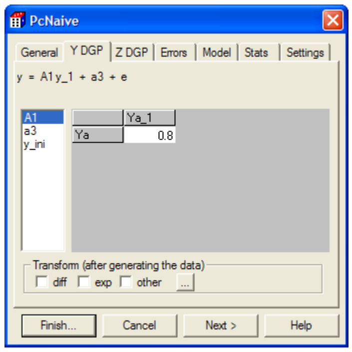
Results – with unit root



As specified, the rejection frequency (size) is 5%, i.e. we reject the true hypothesis 5% of the time.

(3) Introducing a non-unit root



Autoregressive parameter, A1 = 0.8

Intercept, a3 = 0

Initial value, y_ini = 0

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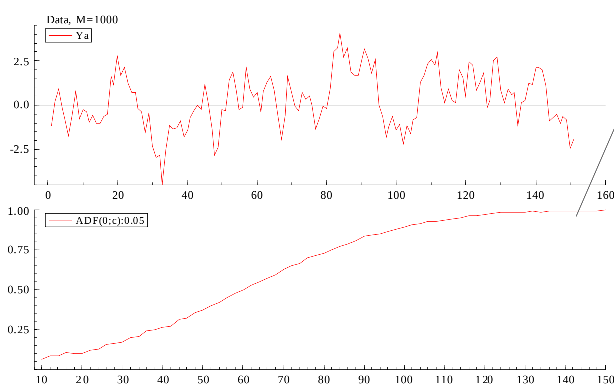
Results – without unit root



Note that it takes many observations before we reject the false null 100% of the time. This illustrates the power of the test.

Look at a sub-sample of the data and it is easy to see why you might accept the null of a unit root.

(4) The importance of power

- If a test is not very powerful, we might be accepting a false hypothesis.
- It is important to bear this in mind when using the DF test.
- You might "find" a unit root, when none is present!

9.2 The impact of un-modelled structural breaks

Now we want to analyze the effect on the DF test of a break in the mean of y_t. To do that we consider a DGP given by

$$y_t = 0.75 \cdot \mathsf{I}(t \geq 75) + \theta \cdot y_{t-1} + u_t, \quad u_t \sim N(0,1), \tag{9.3}$$

for $t = 1, ..., 150$. Here $\mathsf{I}(\cdot)$ is the indicator function that takes the value one if the expression is brackets is true and zero otherwise, i.e.

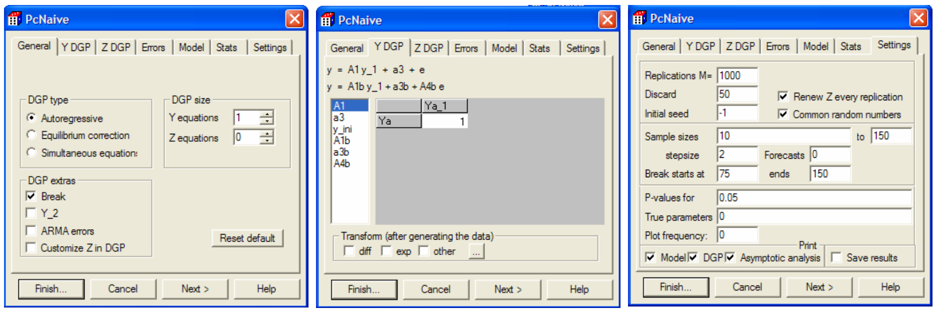
$$\mathsf{I}(t \geq 75) = \begin{cases} 1 \text{ if } t \geq 75 \\ 0 \text{ otherwise} \end{cases}.$$

Note that the DGP implies that the constant term in the model changes. More precisely, y_t is generated with a zero constant term for $t = 1, 2, ..., 74$ and with a constant term of 0.75 for $t = 75, 76, ..., 150$. We consider a regression model that neglects the change in the mean,

$$y_t = \beta_0 + \beta_1 \cdot y_{t-1} + \epsilon_t, \tag{9.4}$$

and look again on the DF $t-$test.

(1) How to specify the DGP



Autoregressive parameter, A1 = 1

Intercept, a3 = 0

Initial value, y_ini = 0

After break, A1b = 1

After break, a3b = 0.75

A4b should also be set to 1

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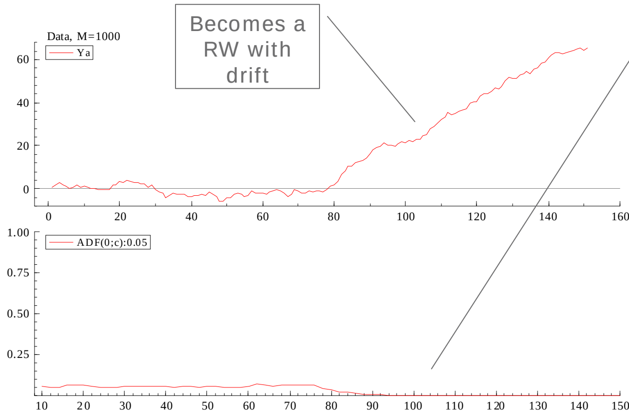
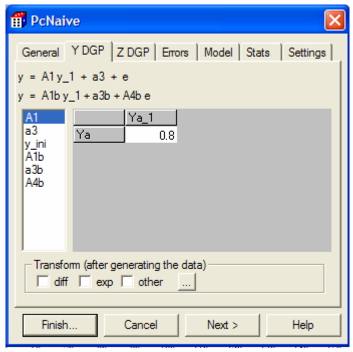
Results – with unit root and a break at T=75

Becomes a RW with drift



Note that the rejection frequency drops to 0 after t = 75. Not very good, but we can live with it!

(2) How to specify the DGP



Autoregressive parameter, A1 = 0.8

Intercept, a3 = 0

Initial value, y_ini = 0

After break, A1b = 0.8

After break, a3b = 0.75

A4b should also be set to 1

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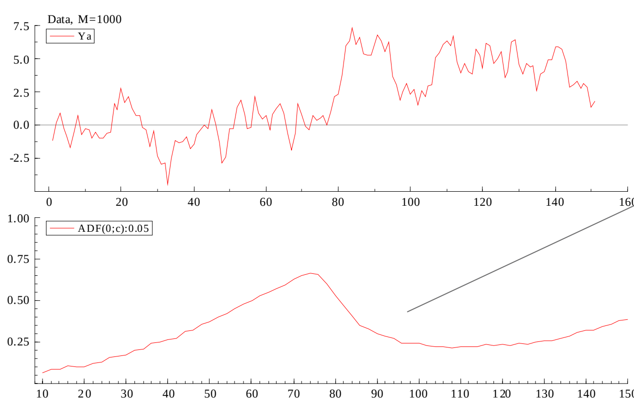
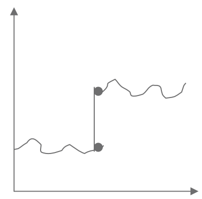
Results – without unit root and a break at T=75



Note how the power of the test drops away after t = 75.
This is a problem!

(3) Impact of un-modelled structural breaks

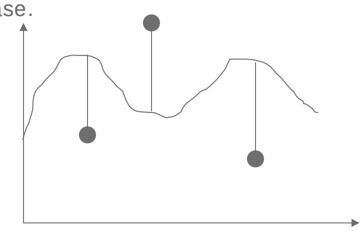
- Un-modelled breaks have an impact on the size and the power of the DF test.
- In particular, the power of the test breaks down, so we accept the false null and find a unit root when it is not present.
- The intuition is that DF tests whether shocks have a permanent effect. A structural break is a shock which impacts on every period afterwards.
- Another way of thinking about it: A unit root implies that the process follows a random walk. This is perhaps the only way (given the restrictions of the model) of modelling a process that looks like this:



(4) How to model a break

- Obvious solution is to add a dummy (= 0 for every period up to the break, = 1 for every period after the break).
- However, this changes the DF distribution (shifts it even further to the left).

- **Note:** In what we have just looked at, a break gives the impression of non-stationarity. An alternative problem (which we will look at later) is that many large outliers (e.g. because *Statistics Denmark* typed the wrong number) might give the impression of stationarity when this is not the case.



9.3 Unit root testing: Important concepts

Below is a list of statements on the Dickey-Fuller unit root test. Comment on each of the statements.

(1) "The null hypothesis of a DF test is stationarity".

(2) "The Dickey-Fuller distribution is only different from a $N(0,1)$ in small samples. For $T \to \infty$, the distribution of the DF $t-$test will approach a standard normal".

(3) "Graphically, a stochastic trend, $y_t = \sum_{i=1}^{t} \epsilon_i$, looks very much like a deterministic trend, $x_t = \mu \cdot t$. Particularly in large samples".

(4) "The asymptotic distribution of the DF $t-$test depends on the determinitic specification of the regression model, i.e. whether a constant or a trend term is included".

(5) "The asymptotic distribution of the augmented DF $t-$test depends on the number of lags included in the regression model".

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Next time

- We (hopefully!) start on cointegration, looking at a full exam question (i.e. parts a-c).
- You will choose a cointegrating relation from *Mona* to look at in class. You have to answer a few questions and present your answers.
- It is therefore a good idea to print Chapter II from *Mona – a quarterly model of the Danish economy* and read it before you come next week. ***Remember to bring it with you.*** It is available on *Nationalbankens* homepage (in Danish and English).